A Compact Dual-Band Branch-Line Coupler Based on the Interdigital Transmission Line

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Abstract: A dual-band (DB) microstrip branch-line coupler with quadrature phase difference based on the interdigital transmission line (TL) is presented. Interdigital TLs are realized by cascading interdigital capacitors (cells of the structure) in series. The design procedure to reach the dual-band interdigital transmission lines at arbitrary frequencies is presented. Dual-band operation for branch-line coupler is achieved by implementing the interdigital TLs with 3 cells in each branch to provide a phase of \(-90^\circ\) at the lower operation frequency band (2GHz) and \(-270^\circ\) phase shift at the upper frequency band (3GHz). Branches length is approximately \(0.8 \lambda_g\) and as compared to the conventional DB composite right/left handed (CRLH) branch-line couplers, the proposed structure achieves smaller dimension. Also, simulation results show that the dual-band proposed coupler exhibits an amplitude balance of \(\pm 2^\circ\) from 1.9 GHz to 2.2 GHz in the lower band and from 2.8 GHz to 3.2 GHz in the upper band. Moreover, at mentioned frequency bands, the phase difference between the coupled and through ports is \(90^\circ \pm 10^\circ\).

Keywords: Interdigital capacitor, branch-line coupler, dual-band component, composite right/left handed (CRLH).

1. Introduction

Couplers are indispensable components in radio frequency (RF)/microwave communication systems which are utilized in a variety of circuits including modulators, balanced amplifiers, balanced mixer, and phase shifters [1]. Among them, branch-line couplers (quadrature hybrids) are 3-dB directional couplers with a \(90^\circ\) phase difference in outputs of the through and coupled ports. This type of hybrids is often made in microstrip or stripline forms [1].

Dual-band (DB) components are devices that exhibit certain functionality at two different frequencies. Such devices are of interest for modern microwave and wireless communication systems because they make possible operation at two different bands without the need to design two different mono-band (MB) circuits [2].

In the past few years, there has been a great interest in the field of metamaterials (MTMs), especially composite right/left handed (CRLH) structures (interdigital/stub or SMT/microstrip structures), and the microwave circuits based on the unusual properties of them [2]. Contrary to conventional (right-handed) transmission lines, which are intrinsically MB structures, CRLH transmission lines can exhibit a DB behavior [2].

Interdigital capacitor is a multifinger periodic structure, which uses the capacitance that occurs across a narrow gap between thin-film conductors [3]. In this paper a new transmission line (TL) based on the cascaded interdigital capacitors is introduced. In the other words, if some interdigital capacitors are cascaded to each other, the interdigital TL will be realized. According to the interdigital TL structure, this TL can be considered as a CRLH structure that contains only interdigital capacitors without stubs. So, this TL can show some CRLH structure properties such as DB behavior.

In this paper, a new compact dual-band branch-line coupler is presented. The proposed operation and design principles are based on the interdigital TLs to improve size of the conventional microstrip CRLH dual-band branch-line couplers. Each interdigital TL of the proposed DB branch-line coupler consists of three interdigital capacitors as its cells. Also, interdigital TLs have non-linear phase responses with respect to frequency. These unique properties result from the existence of a series capacitance \(C_{\text{out}}\) in addition to a series inductance \(L\) and a shunt capacitance \(C\), in the equivalent circuit model of an interdigital capacitor structure. Compared with the conventional CRLH DB branch-line couplers [4]-[11], the proposed structure has more attainable size.
In this paper at first, theoretical description and principle of the interdigital TL and its dual-band property is presented. Also, proposed microstrip DB branch-line coupler is introduced. Finally, simulation results of the proposed DB branch-line coupler are demonstrated by using an electromagnetic simulator (Agilent ADS).

2. Interdigital TL

An interdigital capacitor is a multifinger periodic structure which can be used as a series capacitor in microstrip transmission lines technology. This capacitor uses the capacitance that occurs across a narrow gap between thin-film conductors [3]. Fig. 1(a) shows an interdigital capacitor and its equivalent circuit model. As seen in this figure, an interdigital capacitor is made of some gaps. The gap meanders back and forth in a rectangular area forming two sets of fingers that are interdigital. These gaps are essentially very long and folded to use a small amount of area. By using a long gap in a small area, compact single-layer small-value series capacitors can be realized. Typically, values range from 0.05 pF to about 0.5 pF. The capacitance can be increased by increasing the number of fingers, or by using a thin layer of high dielectric constant material such as a ferroelectric between the conductors and the substrate [3]. Series capacitance of an interdigital capacitor, i.e. $C_{\text{int}}$, with physical parameters which have been presented in Fig. 1(a) is equal to:

$$C_{\text{int}} = \frac{\varepsilon_\text{r} + 1}{W} \times \left( N - 3 \right) A_1 + A_2$$

(1)

where $\varepsilon_\text{r}$ is permittivity of the microstrip substrate and $N$ is the number of structure fingers. Approximation expressions for $A_1$ and $A_2$ are obtained by curve fitting the data given in [3]. These expressions are as:

$$A_1 = 4.409 \tan (0.55 \frac{d}{W}) \times \times 10^{-6} \ (pF/\mu m)$$

$$A_2 = 9.92 \tan (0.52 \frac{h}{W}) \times \times 10^{-6} \ (pF/\mu m)$$

(2)

In Fig. 1(a), $L$ and $C$ in equivalent circuit mode are conventional series inductance and shunt capacitance in microstrip TL and are considered as parasitic elements in interdigital structure. Values of these elements can be calculated from TL theory using the length of the structure, i.e. $\ell_{\text{int}}$, as:

$$L = \frac{\varepsilon_{\text{re}} Z_0}{c} \ell_{\text{int}}$$

$$C = \frac{\varepsilon_{\text{re}}}{Z_0 c} \ell_{\text{int}}$$

(3)

where $\varepsilon_{\text{re}}$ is effective permittivity of the microstrip TL whose strip width is $W$, $Z_0$ is characteristic impedance of a microstrip TL with strip width of $W$ ($= (2N - 1)S + 2NW$) and $c$ is the velocity of light in free space. As it is seen in Fig. 1(b), an interdigital TL can be constructed by cascading some interdigital capacitors, which are unit cells of the interdigital TL. Now, for this new transmission line the parameters can be derived as the following. If we define per-unit-length impedance ($Z'$) and admittance ($Y'$) as:

$$Z' = j \left( \alpha L' - \frac{1}{\alpha C'_{\text{int}}} \right)$$

$$Y' = j \alpha C'$$

(4)

where

$$L = L' \ell_{\text{int}}, \quad C = C' \ell_{\text{int}}, \quad C_{\text{int}} = C_{\text{int}} / \ell_{\text{int}}$$

(5)

It is well known from transmission line theory that propagation constant $\gamma$ and characteristic impedance $Z_c$ of a TL with series impedance $Z'$ and parallel admittance $Y'$, is obtained from following equations [1]:

$$\gamma = \sqrt{Z' Y'}$$

(6)

$$Z_c = \frac{Z'}{\sqrt{Y'}}$$

(7)

So for interdigital TL, the complex propagation constant and characteristic impedance are:

$$\gamma_{\text{int}} = j \sqrt{\left( \frac{\alpha L' - 1}{\alpha C'_{\text{int}}} \right) \left( \alpha C' \right)} = j \sqrt{\alpha^2 L' C' - \frac{C_{\text{int}}}{\ell_{\text{int}}}} = j \beta_{\text{int}}$$

(8)

$$Z_{c,\text{int}} = \frac{\sqrt{\left( j \left( \frac{\alpha L' - 1}{\alpha C'_{\text{int}}} \right) \left( \alpha C' \right) \right)}}{\sqrt{\alpha C'}} = \frac{L' \ell_{\text{int}}}{\sqrt{C' \alpha^2 C'_{\text{int}}}}$$

(9)
It is clear from the above equations that $\mathcal{J}_{\text{int}}$ and $Z_{c,\text{int}}$ are real for $\omega > \omega_c = \frac{1}{\sqrt{LC_{\text{int}}}}$.

### 3. Dual-Band Interdigital TL Description

A dual-band (DB) device is a component accomplishing the same function at two different arbitrary frequencies $\omega_1$ and $\omega_2$. Such a component is therefore constituted of TL sections inducing equivalent phase shifts $\phi_1 = \beta_1 l$ and $\phi_2 = \beta_2 l$, where $l$ is the length of the TL sections, at these two frequencies. In other words, a DB component should exhibit a dispersion relation $\beta(\omega)$ satisfying the double condition

$$\beta(\omega = \omega_1) = \beta_1$$

$$\beta(\omega = \omega_2) = \beta_2$$

In this section it will be shown that an interdigital TL can exhibit similar properties of a dual-band component. Let us consider the case of the interdigital TL, for which the propagation constant $\beta_{\text{int}}$ and characteristic impedance $Z_{\text{int}}$ are given by (8) and (9), respectively. For indicated propagation constants $\beta_1$ and $\beta_2$ for required operation at frequencies $\omega_1$ and $\omega_2$, the following conditions must be satisfied:

$$\beta_{\text{int}}(\omega = \omega_1) = \beta_1$$

$$\beta_{\text{int}}(\omega = \omega_2) = \beta_2$$

Thus, insertion of Eq. (11) into Eq. (8) yields a linear system of two equations with the three unknowns $L'$, $C'$, $C'_{\text{int}}$, and there is therefore one available degree of freedom that can be exploited to satisfy matching to terminations of impedance $Z_c$. However, we need only one equation whereas satisfying impedance matching condition at both frequencies results two equations. This problem can be eliminated by applying impedance matching condition at frequency $\omega_c = (\omega_1 + \omega_2)/2$ which leads to a good matching for both two frequencies $\omega_1$ and $\omega_2$.

$$Z_{c,\text{int}}(\omega = \omega_c) = Z_t$$

(12)

From the resulting system of three equations, i.e. Eqs. (10) and (11), after some straightforward calculations, the DB interdigital parameters can be obtained as:

$$C' = \frac{\beta_1^2(\omega_1^2 - \omega_c^2) + \beta_2^2(\omega_2^2 - \omega_c^2)}{2N\omega^2 Z_c(\omega_2^2 - \omega_1^2)}$$

(13)

$$L' = t' \times \frac{1}{C'}$$

$$C'_{\text{int}} = \frac{NC'}{(\omega_c^2 t^2 - \phi_1^2)}$$

where

$$t' = \frac{\beta_2^2 - \beta_1^2}{\omega_2^2 - \omega_1^2}$$

(14)

An artificial interdigital TL of physical length $\ell$ is constituted by the repetition of $N$ unit cells (interdigital capacitors) of length $\ell_{\text{int}}$, so that:

$$\ell = N\ell_{\text{int}}$$

(15)

Because, each cell induces a phase shift of $\Delta \phi$, the total phase shift along the line is then:

$$\phi = N\Delta \phi$$

(16)

According to this relation, the relation $\beta = -\phi/\ell$ and the fundamental substitutions, i.e.,

$$L = L'\ell_{\text{int}}, \quad C = C'\ell_{\text{int}}, \quad C_{\text{int}} = C'_{\text{int}}/\ell_{\text{int}}$$

(17)

the DB interdigital TL unit cell parameters, i.e. parameters of each interdigital capacitor, are obtained as:

$$C = \frac{\phi_1^2(\omega_1^2 - \omega_c^2) + \phi_2^2(\omega_2^2 - \omega_c^2)}{2N\omega^2 Z_c(\omega_2^2 - \omega_1^2)}$$

(18)

$$L = t \times \frac{1}{C}$$

$$C_{\text{int}} = \frac{NC}{(\omega_c^2 t - \phi_1^2)}$$

where

$$t = \frac{\phi_2^2 - \phi_1^2}{\omega_2^2 - \omega_1^2}$$

(19)

It is seen from (18) that for positive values of $L, C, C_{\text{int}}$, the following conditions must be satisfied:

$$\omega_2 > \omega_c \Rightarrow \phi_2 > \phi_1$$

$$\left(\frac{\phi_1}{\phi_2}\right)^2 - \left(\frac{\omega_2}{\omega_1}\right)^2 > 2$$

(20)

### 4. Dual-Band Branch-Line Coupler

A conventional branch-line coupler is a 3-dB directional coupler with a 90° phase difference between the outputs of the through and coupled arms [1] and a dual-band branch-line coupler shows these properties at two frequencies.

Layout of the proposed dual-band branch-line coupler based on the interdigital TLs is shown in Fig. 2, whereas the conventional quarter-wave microstrip lines of the branch-line coupler have been replaced by the interdigital TLs with 3 cells. These dual-band TLs have been introduced in the previous section. In designed structure, the substrate has been chosen to be FR4 with the relative permittivity of 4.8, tan $\delta$ = 0.01 and the thickness is set as 1.6 mm. The central dual-band frequencies are designed to be 2 GHz and 3 GHz. The dimension of the interdigital TLs unit cells used in the horizontal arms of proposed DB branch-line coupler are listed below. $W = 0.6$ mm, $S = 0.2$ mm, $\ell_{\text{int}} = 19$ mm, while for the units in the vertical arms, we have $W = 0.3$ mm, $S = 0.2$ mm, $\ell_{\text{int}} = 21$ mm. The number of fingers in each cell (interdigital capacitor) in both horizontal and vertical arms is 3. This difference between
the dimensions of the units in the horizontal and vertical arm positions is due to the requirements of the impedance and S-matrix for the four-port branch-line coupler [1]. As mentioned before, the nonlinear phase response of the interdigital TL can be used to reduce the dimension of components. In Fig. 2, total lengths of the horizontal and vertical arms are $L_1 = 57\, \text{mm}$ and $L_2 = 63\, \text{mm}$, respectively. So, the size of the largest arm of the coupler is about $0.8\lambda_g$, where $\lambda_g$ is the guide wavelength at lower frequency band, i.e. 2 GHz. According to this thread, the size of the novel dual-band branch-line coupler has been reduced in comparison with the size of the dual-band branch-line couplers presented in [4]-[11]. For example, the size of the branches of the dual-band branch-line couplers which have been presented in [4] and [10] are nearly $0.9\lambda_g$ and $1.6\lambda_g$, respectively.

Simulation results of the proposed dual-band branch-line coupler are demonstrated by using Agilent ADS simulator and are presented in Fig. 3. As these results show in the figure, a dual-band branch-line coupler with nearly 20% bandwidth in the lower and upper frequency bands, around the design frequencies 2 GHz and 3 GHz has been achieved. The amplitudes of $|S_{12}|$ and $|S_{13}|$ are close to -3 dB at central frequencies, while the amplitudes of $|S_{11}|$ and $|S_{14}|$ are less than -20 dB, which proofs the good performance of the designed coupler at desired frequencies. Moreover, Fig. 4 illustrates the phase difference between ports 2 and 3 of the coupler, i.e. through and coupler ports. This phase difference is $\pm 90^\circ$ at design frequencies and exhibits a phase-balance ($\pm 10^\circ$) bandwidth from 1.9 GHz to 2.1 GHz and 2.8 GHz to 3.2 GHz for lower and upper frequency bands, respectively.

![Fig. 2: Layout of the proposed dual-band branch-line coupler](image)

![Fig. 3: Simulation results of the proposed DB branch-line coupler](image)

![Fig. 4: Simulated phase difference between the through and coupled ports for the proposed coupler of Fig. 2.](image)

5. Conclusion

A new compact dual-band branch-line coupler based on the interdigital TLs has been presented in this paper. Used interdigital TLs in coupler structure have been realized with cascading three interdigital capacitors to each other. The design procedure to reach the dual-band interdigital TLs at two arbitrary frequencies was also presented. The proposed dual-band branch-line coupler exhibits the amplitude balance of $\pm 2\, \text{dB}$ and the phase balance of $\pm 10^\circ$ from 1.9 GHz to 2.1 GHz and 2.8 GHz to 3.2 GHz for lower and upper frequency bands. The device is also smaller than the previously proposed dual-band branch-line couplers implemented by means of composite right/left handed (CRLH) transmission lines.

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References