3D meshless simulation of laser ray deviation under thermal lensing effect

Arman Afsari*†, Abbasali Heidari and Masoud Movahhedi

Faculty of Electrical and Computer Engineering, Yazd University, Yazd, Iran

SUMMARY

In this study, one of the most important destructive effects in end-pumped Nd:YAG laser, that is, thermal lensing is analyzed. Different partial differential equations, supplemented by suitable boundary conditions, are solved and joined to illustrate the fluctuations in laser Transverse Electromagnetic (TEM) mode. Because of no analytic demonstration of this effect, an efficient meshless method known as radial point interpolation meshless method (RPIM) is used to simulate the problem divided into two two-dimensional cases, that is, xz and yz planes so as to be graphically understandable. The results are in good agreement with measurement reports, and the RPIM shows better accuracy than FEM. Copyright © 2015 John Wiley & Sons, Ltd.

Received 3 October 2014; Revised 17 March 2015; Accepted 1 June 2015

KEY WORDS: meshless method; thermal lensing effect; end-pumped lasers; heat equation; refractive index; TEM mode; meshless method

1. INTRODUCTION

Interest has increased in the past years in using semiconductor diode lasers to excite solid state lasers based on rare-earth ion-doped transparent solids such as neodymium-doped yttrium aluminum garnet (Nd:YAG). Traditionally, these solid state lasers are excited by flashlamps that emit broadband radiation. Lamp-pumped systems are inefficient, however, with typically 1% electrical-to-optical efficiency, and the lamps need replacement after approximately 200 h when operated continuously. Diode laser pump sources allow operation at higher efficiency (10%) and longer life (20,000 h).

One of the critical effects in laser physics and engineering is the ray deviation in active medium and laser core. This phenomenon has appeared even if the end mirrors are kept in good cooling conditions such that they do not encounter any deformations. More precisely, when the laser rod, as active medium of laser, is warmed because of pumping ray, the refractive index of the rod (as an open cavity) is changed. Different values for refractive index in the rod cross section will result in unwanted phase fluctuations that in turn cause the nonuniform wave distribution along the cavity. Therefore, the TEM\textsubscript{00} mode (as the dominant mode in laser open cavity) deviates and faces with some fluctuations [1—4].

Moreover, the thermal lensing effect depends on different parameters, and taking them into account makes it impossible to find a general analytic solution for the effect. Indeed, different materials follow different deformation patterns and different relations between heat distribution and refractive index. However, some specific materials are of paramount importance in laser theory, for example, Nd:YAG. This importance leads us toward accumulating all the governing equations on this material so as to study the thermal lensing effect, accurately. Hence, because of the popularity of end-pumped Nd:YAG lasers, figuring out the behavior of stimulated rays under aforementioned effect helps us design and optimize this kind of laser.

*Correspondence to: Arman Afsari, Faculty of Electrical and Computer Engineering, Yazd University, Yazd, Iran.
†E-mail: afsari@yazd.ac.ir

Copyright © 2015 John Wiley & Sons, Ltd
As mentioned, the quantitative determination of the effective focal length due to thermal effects in end-pumped lasers is difficult compared with lamp-pumped because of the small size involved and also because of the higher sensitivity required. Several different approaches have been used including those based on interferometry, analysis of the output beam parameters, transverse mode beat frequency, and degeneration in the resonator length. Nevertheless, most of these methods are not accurate enough, and some important features are not well understood. For instance, it is recognized that the center of the rod contributes to thermal lensing more than its edges, because of hotter temperature at the center.

In recent years, simulations of different phenomena in lasers have risen in popularity. Brilliant works based on numerical techniques have been published in accordance with different lasers and applications. Among these works, double quantum well lasers [5], temperature effect on laser sintering process [6], coupled mode-locked laser equations in time domain [7], and the surface-emitting lasers [8], have successfully been simulated.

Among different computational techniques, meshless methods as alternative numerical approaches to eliminate the well-known drawbacks in the finite element and boundary element methods have attracted much attention in the past decade, because of their flexibility, and because of their potential in negating the need for the human-labor intensive process of constructing geometric meshes in a domain governed by partial differential equations. Meshless methods may also alleviate some other problems associated with the finite element method, such as locking and element distortion [9–14]. Some popular meshless techniques are the meshless local Petrov–Galerkin method and radial point interpolation meshless method (RPIM). This paper follows two parallel goals. The first one is to show the advantages of RPIM versus FEM in terms of precision and accuracy. The second goal is to accurately solve an important problem in laser physics, that is, thermal lensing effect for which no analytic solution exists. It is seen that accuracy of RPIM is better than FEM because of the aforementioned reasons, while both methods possess same computational time and complexity, because of their similar mathematical fundamentals [14].

The rest of the paper has been organized as follows. Section 2 investigates the heat equation and heat effect on refractive index of end-pumped Nd:YAG laser through applying the RPIM to solve the given equations. Following this, the TEM mode fluctuations are simulated in accordance with the variations of refractive index (Section 3). The error comparison graphs between RPIM and FEM are seen at the end of each section to show the better accuracy of RPIM. The FEM used in this paper implements first-order polynomial basis functions in tetrahedral mesh configuration. Finally, conclusions are presented in Section 4.

2. THE THERMAL LENSING EFFECT IN END-PUMPED ND:YAG LASERS

Thermal Lens is a crucial effect in laser materials, especially when operating in an end-pumping configuration (due to the much localized heat deposition achieved in this case). In most of the situations, thermal lensing is an undesirable effect that leads to deterioration in the laser output power and/or in the spatial quality of the laser beam. On the other hand, in some configurations, such as microchip designs, this phenomenon is required for stable laser oscillation. In any case, and independently of the geometrical configuration of the laser cavity used, a precise knowledge of the thermo-optical properties of the system, determining the generated heat in the active volume and the induced thermal lensing effect, is very important to laser design.

In this section, the end-pumped Nd:YAG laser with thermal lensing phenomenon is analyzed. First of all, the heat equation is solved. Then, the evaluation of refractive index in Nd:YAG crystal using RPIM is presented and compared with FEM.

2.1. Heat equation and refractive index

The thermal coefficient values of refractive index are in increasing demand in many optical applications. Most of the time, there is no data available in the literature for various laser-active materials. In recent years, because of the rapid improvement in fiber optic technology and sensing devices, and in addition to the many different types of non-fiber optics techniques available for measuring refractive index, fiber optic sensors have become one of the major techniques in measuring refractive index.
Figure 1. The Nd:YAG crystal as laser rod with end mirrors.

Figure 2. Heat distribution in laser rod; unit: Kelvin.

Figure 3. Refractive index over cross section.
At the beginning, the structure of end-pumped Nd:YAG laser-open cavity has been shown in Figure 1. The laser rod lies between two mirrors. The cross section of laser mirrors and rod is a square with a width 0.8 mm and its length is 8 mm. The pumping ray illuminates the center of the rod from left side and is considered as a cylindrical heat source (315.5 Kelvin as a typical value). The pumping ray radius is 0.05 mm along the laser rod. Mirrors and open boundaries are in direct contact with room temperature (293 Kelvin) (these are Dirichlet boundary conditions). Such heat distribution will eventually change the refractive index that is a function of temperature [15].

The governing steady-state thermal equation is the following Laplace equation: [15]

\[ \nabla^2 T = 0 \]  \hspace{1cm} (1)

Using the functional of Laplace equation, the following system of equations is derived:

\[ KT = B, \]  \hspace{1cm} (2)

where the elements of stiffness matrix, that is, \( K_{ij} \), and excitation matrix, that is, \( B_{i1} \), are

\[ K_{ij} = \int_{\Omega} \nabla N_i \cdot \nabla N_j d\Omega, \]  \hspace{1cm} (3)
where $N_i$ is the adaptive basis function proposed in [14] as follows:

$$N_i(x) = \exp\left(-\frac{\pi}{2}(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2\right) \cos\left(\frac{\pi}{2} \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}\right)$$

(4)

and

$$B_i = 0, \quad \text{except Dirichlet boundary values} \quad (5)$$

Setting the boundary values to 293 K and 315.5 K, (2) yields a unique solution. The solution of above heat equation using RPIM is seen in Figure 2.

The refractive index of optical glasses changes with temperature, the extent of which depending on the glass type and on the wavelength. It also changes with air pressure; therefore, in the following, we distinguish between the refractive index relative to vacuum or absolute refractive index and the
refractive index at normal air pressure. All equations given in this paper are at normal air pressure. The standard measurement temperature for refractive index values given on test certificates is 22 °C. By taking into account the temperature coefficients of the Nd:YAG, it is possible to calculate the change of refractive index for any wavelength.

If the temperature of any point in Nd:YAG cross section is $T$ with the corresponding refractive index $r$, and the coolest temperature is $T_0$, associated with the refractive index $r_0$, the variations of refractive index is calculated according to the following ordinary differential equation [16];

$$\frac{dr}{dT} = \frac{r_0^2 - 1}{2r_0} \left( \frac{d_0 + 2d_1 \Delta T + 3d_2 (\Delta T)^2}{\lambda^2 - \lambda_0^2} \right),$$

where $c_0, c_1, d_0, d_1, d_2$ are crystal coefficients, $\Delta T = T - T_0$, $\lambda_0$ and $\lambda$ are the wavelengths in free space and crystal, respectively [16].
Figure 3 shows the solution of (6). As seen, variations of refractive index along the cross section is noticeable. This solution has been calculated when the total number of uniformly distributed nodes is $10^4$. Therefore, the heat effect on refractive index was demonstrated. However, the main problem is yet to be solved. As the last point, it is seen from Figure 4, that the RPIM and FEM are in good agreement with exact solution of heat equation. However, the better accuracy of RPIM with respect to FEM is seen in the next section.

The next section focuses on the laser open cavity simulation, that is, complex eigenvalue problem of Nd:YAG laser so as to study the TEM mode profile under thermal lensing effect.

3. SIMULATION OF TEM PROFILE IN END-PUMPED Nd:YAG LASER USING RADIAL POINT INTERPOLATION MESHLESS METHOD

This section shows the solution of laser complex eigenvalue problem (laser cavity is open from four sides) using RPIM. In this part, the effect of variable refractive index on TEM mode is depicted.
3.1. The eigenvalue problem of laser open cavity

According to [16], to compute the eigenmodes of a 3D open cavity (with standing waves), two traveling electric waves propagating in opposite directions along $z$ axis must be taken into account. The following complex eigenvalue problems show the behavior of above waves.

$$-\nabla^2 u_r + 2j k_0 \frac{\partial u_r}{\partial z} + (k_0^2 - k^2) u_r = k_{pf} u_r \quad \text{on } \Omega$$

(7)

$$-\nabla^2 u_l - 2j k_0 \frac{\partial u_l}{\partial z} + (k_0^2 - k^2) u_l = k_{pf} u_l \quad \text{on } \Omega.$$  

(8)

where $k = rk_0$, $r$ and $k_{pf}$ are the refractive index (obtained before) and the phase fluctuations, respectively. $k_0$ is the wave number of pumping ray; $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $u_r$ and $u_l$ are the right and left going waves, respectively.
The equations are similarly supplemented by the following boundary conditions:

\[
\begin{align*}
    u_r &= -u_l \quad \text{on mirrors} \\
    \frac{\partial u_r}{\partial \zeta} &= \frac{\partial u_l}{\partial \zeta} \quad \text{on mirrors} \\
    \frac{\partial u_r}{\partial \mathbf{s}} &= j k_0 u_r \quad \text{on open boundaries} \\
    \frac{\partial u_l}{\partial \mathbf{s}} &= j k_0 u_l \quad \text{on open boundaries},
\end{align*}
\]

where \(s\) is the normal vector of open boundaries and the total standing electric field is \(E = u_r + u_l\). Using RPIM, the following matrix representation of the complex eigenvalue problem for both equations is obtained

\[
KU = k_{pf} BU
\]

Clearly, \(K\) and \(B\) are \(n \times n\) dimensions, \(U\) is an \(n \times 1\) vector, and a uniform nodal distribution is used. The elements of \(K\) and \(B\) matrices are as follow:

\[
\begin{align*}
    K_{ij} &= \int_{\Omega} \left[ (\nabla N_i, \nabla N_j) + (k_0^2 - k^2) N_i N_j \pm 2 j k_0 \frac{\partial (N_i N_j)}{\partial \zeta} \right] d\Omega - j k_0 \int_S N_i N_j dS \\
    B_{ij} &= \int_S N_i N_j dS,
\end{align*}
\]

where \(\pm\) stands for traveling waves propagating toward right (+) and left (−) sides.

Solving the aforementioned eigenvalue problem, that is, Equation 10 through well-developed computer programs, the cavity TEM mode solution is obtained. Figure 5 shows the complex eigenvalues in polar coordinates. To show the wave behavior and fluctuations, the eigenvalue \(k_{pf} = (0.51 - j 8.57) \times 10^6\) (whose phase is about 270° and amplitude is minimum) is considered. For this complex eigenvalue, the \(u_r\) in vertical and horizontal planes has been depicted in Figures 6 and 7. Figures 8 and 9 illustrate the corresponding \(u_l\). As seen, both traveling waves have nonuniformly been distributed. The vertical and horizontal representations of TEM mode profile, that is, the normalized electric component of TEM mode, \(\left( \frac{|E(x, y, z)|}{|E_{max}|} \right)^2\), is seen in Figures 10 and 11. Finally, Figure 12 shows the TEM profile in an arbitrary cross section with \(z = 0.6\) mm. The RPIM solutions of these 2D simulations of TEM mode are in good agreement with measurement results in [16] as seen in Figure 13 for cross section \(z = 0.6\) mm and \(y = 0\). This agreement reveals the fact that these simulations accurately illustrate the real thermal lensing effect. The error comparison graphs are seen in Figure 14 and confirm the better precision of RPIM versus FEM.
4. CONCLUSION

In this work, the thermal lensing effect in end-pumped Nd:YAG lasers was studied. The effect of pumping ray heat on refractive index changes the index along laser rod cross section. This change, in turn, fluctuates the TEM mode profile as a destructive effect on laser performance. Along the paper, the RPIM as a strong numerical method was used to simulate different effects. RPIM simulation results are in good agreement with measurement records and possess better accuracy than FEM.

ACKNOWLEDGEMENTS

In accomplishing this paper, the authors gratefully acknowledge Prof. Holger Wendland from the Institute of Mathematics, University of Oxford, for his instructions and steerage in area of using radial point interpolation meshless method.

REFERENCES